

$$35. f(x) = x^3 + ax^2 + bx$$

solve for a & b so that $f(x)$ has a
min at $x=3$ and a max at $x=-1$

$$f'(x) = 3x^2 + 2ax + b$$

$$f'(3) = 3(3)^2 + 2a(3) + b$$

Plug in 3

$$f'(-1) = 3(-1)^2 + 2a(-1) + b$$

$$0 = 27 + 6a + (2a - 3)$$

$$0 = 8a + 24$$

$$8a = -24$$

$$a = -3$$

Solve
for
B

$$0 = 3 - 2a + b$$

$$b = 2a - 3$$

Solve for b

$$b = 2a - 3$$

$$b = 2(-3) - 3$$

$$b = -9$$

if $f(x)$ has a min at
 $x=3$ and a max at $x=-1$,

$$f(x) = x^3 - 3x^2 - 9x$$

b) to find inflection points, we use the second derivative test.

$$f(x) = 3x^2 + 2ax + b$$

$$f'(x) = 6x + 2a$$

$$0 = 6(1) + 2a$$

$$2a = -6$$

$$a = -3$$

$$0 = 3(4)^2 + 2(-3)(4) + b$$

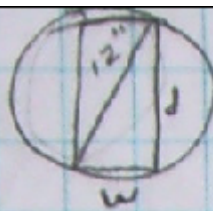
$$0 = 48 - 24 + b$$

$$b = -24$$

if $f(x)$ has a min at $x=1$
and an inflection pt. at $x=4$,

$$f(x) = x^3 - 3x^2 - 24x$$

37. a)



$$S = wd^2$$

$$144 = w^2 + d^2$$

$$d^2 = 144 - w^2$$

$$144 = (\sqrt{48})^2 + d^2$$

$$144 = 48 + d^2$$

$$d^2 = 96$$

$$\boxed{d = \sqrt{96}}$$

$$\boxed{d = 9.79}$$

$$S = w(144 - w^2)$$

$$S = 144w - w^3$$

$$S' = 144 - 3w^2$$

$$S = \sqrt{48} \cdot (\sqrt{96})^2$$

$$S = \sqrt{48} \cdot 96$$

$$\boxed{S = 665.11}$$

$$0 = 144 - 3w^2$$

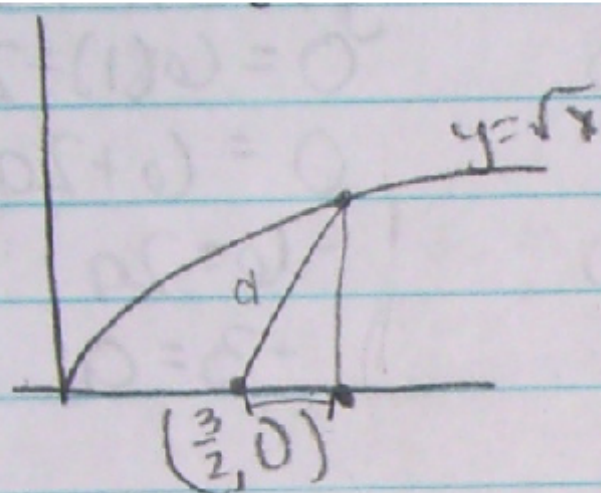
$$3w^2 = 144$$

$$w^2 = 48$$

$$\boxed{w = \sqrt{48}}$$

$$\boxed{w = 6.93}$$

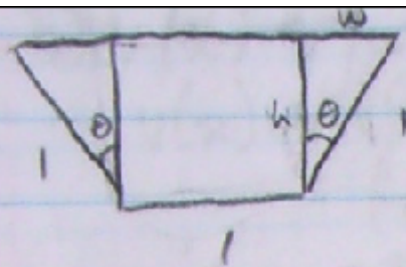
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$$\begin{aligned}
 d^2 &= \left(x - \frac{3}{2}\right)^2 + x^2 \\
 d &= \sqrt{\left(x - \frac{3}{2}\right)^2 + x^2} \\
 d' &= \left(2\left(x - \frac{3}{2}\right)\right)(1) + 1x \\
 &= 2x - 3 + 1 \\
 0 &= 2x - 2 \\
 2 &= 2x \\
 1 &= x
 \end{aligned}$$

$$\begin{aligned}
 d &= \sqrt{\left(1 - \frac{3}{2}\right)^2 + 1^2} \\
 &= \sqrt{-\frac{1}{2}^2 + 1} \\
 &= \sqrt{1.25} \\
 &\approx 1.12 = \frac{\sqrt{5}}{2}
 \end{aligned}$$

47.



$$\cos \theta = \frac{h}{1}$$

$$\cos \theta = h$$

$$\sin \theta = \frac{w}{1}$$

$$\sin \theta = w$$

$$A = \cos \theta \sin \theta + \cos \theta$$

$$A' = \cos \theta \cos \theta + \sin \theta - \sin \theta - \sin \theta$$

$$= \cos^2 \theta - \sin \theta - \sin \theta$$

$$= 1 - \sin^2 \theta - \sin^2 \theta - \sin \theta$$

$$0 = -2\sin^2 \theta - \sin \theta + 1$$

$$\theta = \frac{1 \pm \sqrt{1+8}}{-4}$$

$$\theta = \frac{1 \pm 3}{-4}$$

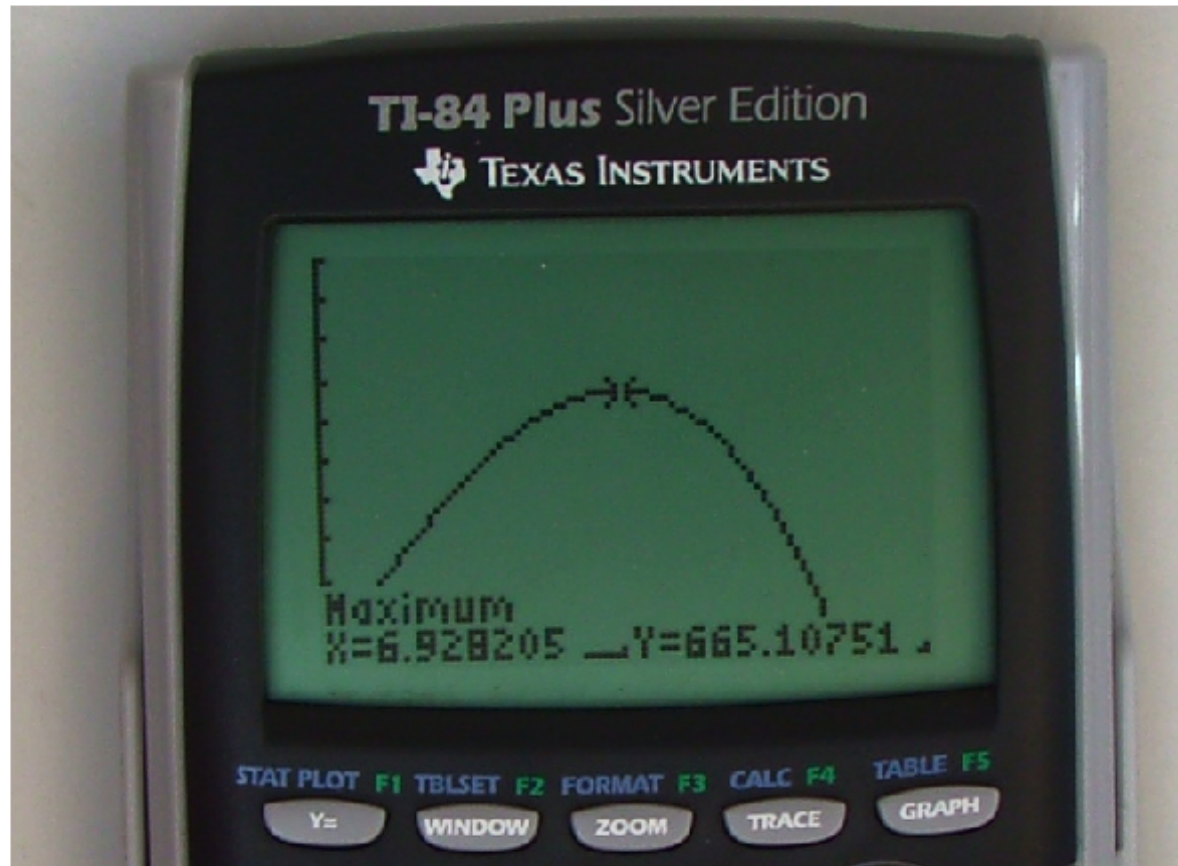
$$\theta = -1, \frac{1}{2}$$

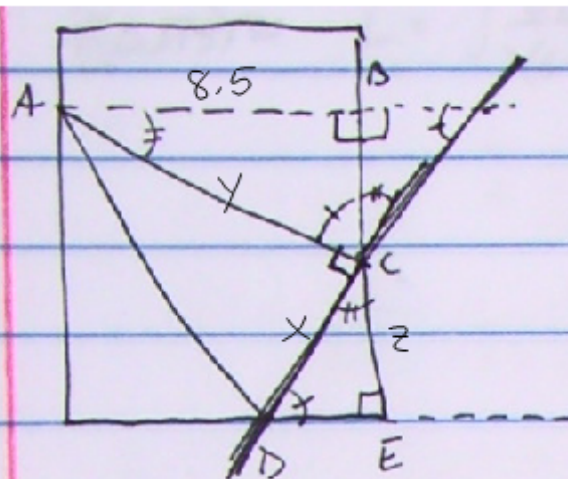
$$\sin \theta = -1, \frac{1}{2}$$

$$\sin^{-1}(-1) = -\frac{\pi}{2}$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} = 30^\circ$$





$$\frac{y}{8.5} = \frac{x}{7}$$

$$\frac{y}{8.5} = \frac{x}{\sqrt{8.5(2x-8.5)}}$$

$$\frac{y^2}{8.5^2} = \frac{x^2}{8.5(2x-8.5)}$$

$$y^2 = \frac{x^2 \cdot 8.5}{(2x - 8.5)}$$

$$L^2 = x^2 + \frac{x^2 \cdot 8.5}{2x - 8.5} \rightarrow L^2 = \frac{2x^3 - 8.5x^2 + 8.5x^2}{2x - 8.5}$$

$$L^2 = \frac{2 \times 3}{2 \times -8.5}$$

$$b) \quad \frac{L^2 = 2x^3}{(2x - 8.5)}$$

$$L' = \frac{(2x - 8.5)(6x^2) - (2x^3)(2)}{(2x - 8.5)^2}$$

$$= \frac{12x^3 - 51x^2 - 4x^3}{(2x - 8.5)^2}$$

$$= \frac{8x^3 - 51x^2}{(2x - 8.5)^2}$$

$$0 = \frac{x^2(8x - 51)}{(2x - 8.5)^2}$$

$$0 = 8x - 51$$

$$51 = 8x$$

$$\frac{51}{8}$$

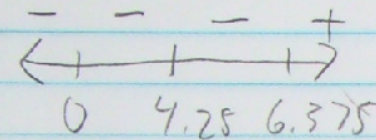
$$0 = 2x - 8.5$$

$$\frac{8.5}{2} = \frac{2x}{2}$$

$$x = 0$$

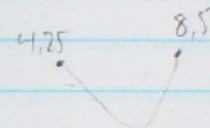
$$x = 6.375$$

$$4.25 = x$$



$$6.375 = \min$$

$$c) \quad \boxed{8.5 = \max}$$



$$f(6.375) = 11.042$$

$$r^2 + h^2 = 16 \Rightarrow r^2 = 16 - h^2$$

$$\frac{1}{3}\pi r^2 \cdot h$$

$$\frac{1}{3}\pi(16 - h^2) \cdot h \Rightarrow \left(\frac{16}{3}\pi - \frac{1}{3}\pi h^2\right) \cdot h \Rightarrow \frac{16}{3}\pi h - \frac{1}{3}\pi h^3$$

$$f(x) = \frac{16}{3}\pi h - \frac{1}{3}\pi h^3$$

$$f'(x) = \frac{16}{3}\pi - \pi h^2$$

$$f'(x) = \pi\left(\frac{16}{3} - h^2\right)$$

$$f'(x) = 0 \text{ at } h = \pm\sqrt{16/3}$$

Volume is maximized when $h = \sqrt{16/3}$ (can't have a negative length)

$$r^2 = 16 - \sqrt{16/3}^2 \Rightarrow 16 - 16/3 \Rightarrow 32/3 \quad r = \sqrt{32/3}$$

$$\text{Circumference} = 2\pi r$$

$$\text{Original Cone} = 2\pi(4) = 8\pi$$

$$\text{Cone} = 2\pi(\sqrt{32/3})$$

$$x = 8\pi - 2\pi(\sqrt{32/3}) = 4.61193 \text{ inches} = x$$